



MEASUREMENT AND  
ERRORS

AQA AS LEVEL

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# AQA Physics A-Level

## Section 1: Measurements and their Errors Notes

### 3.1 Measurements and their errors

#### 3.1.1 - Uses of SI units and their prefixes

##### Measurement of base Quantities:

It involves two steps:

- Choice of standard
- Procedure compares quantity to standard, determines number and unit as measurement.

##### System of Units:

Before the development of SI units, three systems of units were commonly used:

“International standard unit used for measuring physical quantities, ensuring consistency and uniformity in scientific and technical measurements worldwide.”

| System of Unit | Fundamental units                       |
|----------------|---|
| CGS system     | Centimeter (cm)<br>Gram(g)<br>Second(s) |
| F.P.S system   | Foot(ft)<br>Pound(lb)<br>Second(s)      |
| M.K.S system   | Metre(m)<br>Kilogramme(kg)<br>Second(s) |

Why we need SI units?

- Standardization
- Scientific research
- Engineering and industry
- International trade
- Education

## Base quantities:

Fundamental quantities that serve as the basis for all other measurements.

## Base units:

“The units associated with the base quantities are called base units.”

| SI Base Units             |                        |           |        |
|---------------------------|------------------------|-----------|--------|
| Base quantity             |                        | Base unit |        |
| Name                      | Typical symbol         | Name      | Symbol |
| time                      | $t$                    | second    | s      |
| length                    | $l, x, r, \text{etc.}$ | meter     | m      |
| mass                      | $m$                    | kilogram  | kg     |
| electric current          | $I, i$                 | ampere    | A      |
| thermodynamic temperature | $T$                    | kelvin    | K      |
| amount of substance       | $n$                    | mole      | mol    |
| luminous intensity        | $I_v$                  | candela   | cd     |

The SI units of quantities can be **derived** by their equation, e.g.

The diagram shows the equation  $F = ma$ . The letter  $F$  is blue and labeled 'Vector' and 'Force (N)'. The equals sign is green. The letter  $m$  is red and labeled 'Scalar' and 'mass (kg)'. The letter  $a$  is yellow and labeled 'Vector' and 'acceleration ( $m/s^2$ )'. Arrows point from the labels to the corresponding letters.

$$V = \frac{\text{potential energy}}{\text{charge}} = \frac{N \cdot m}{C} = \frac{kg \cdot m^2}{A \cdot s^3}$$

$$= kgms^{-2} \text{ (Also known as N)}$$

### Prefixes:

“Prefixes in physics modify units to indicate larger or smaller values”.

| Prefix | Label | Decimal Value                     | Scientific | Colloquial    |
|--------|-------|-----------------------------------|------------|---------------|
| yocto  | y     | 0.000 000 000 000 000 000 000 001 | $10^{-24}$ | septillionth  |
| zepto  | z     | 0.000 000 000 000 000 000 001     | $10^{-21}$ | sextillionth  |
| atto   | a     | 0.000 000 000 000 000 001         | $10^{-18}$ | quintillionth |
| femto  | f     | 0.000 000 000 000 001             | $10^{-15}$ | quadrillionth |
| pico   | p     | 0.000 000 000 001                 | $10^{-12}$ | trillionth    |
| nano   | n     | 0.000 000 001                     | $10^{-9}$  | billionth     |
| micro  | $\mu$ | 0.000 001                         | $10^{-6}$  | millionth     |
| milli  | m     | 0.001                             | $10^{-3}$  | thousandth    |
| centi  | c     | 0.01                              | $10^{-2}$  | hundredth     |
| deci   | d     | 0.1                               | $10^{-1}$  | tenth         |
| ---    | ---   | 1                                 | $10^0$     | one           |
| deka   | da    | 10                                | $10^1$     | ten           |
| hecto  | h     | 100                               | $10^2$     | hundred       |
| kilo   | k     | 1 000                             | $10^3$     | thousand      |
| mega   | M     | 1 000 000                         | $10^6$     | million       |
| giga   | G     | 1 000 000 000                     | $10^9$     | billion       |
| tera   | T     | 1 000 000 000 000                 | $10^{12}$  | trillion      |
| peta   | P     | 1 000 000 000 000 000             | $10^{15}$  | quadrillion   |
| exa    | E     | 1 000 000 000 000 000 000         | $10^{18}$  | quintillion   |
| zetta  | Z     | 1 000 000 000 000 000 000 000     | $10^{21}$  | sextillion    |

#### Prefixes and their relationship to “whole values”

### Examples:

$6\mu\text{F}$  (micro farads) =  $6 \times 10^{-6}\text{F}$

$10\text{G}\Omega$ (giga ohm) =  $10 \times 10^9\Omega$

| Prefix                       | Symbol | Move decimal pt. | Value |
|------------------------------|--------|------------------|-------|
| Mega= <u>1,000,000</u>       | M      | + 6 places       | 1 M   |
| Kilo= <u>1,000</u>           | K      | + 3 places       | 1 K   |
| “whole values”               | Amp    | Volt             | Ohm   |
|                              | Farad  | Hertz (C.P.S.)   |       |
| milli= <u>0.001</u>          | m      | - 3 places       | 1 m   |
| micro= <u>0.000001</u>       | u      | - 6 places       | 1 u   |
| nano= <u>0. 000000001</u>    | n      | -9 places        | 1n    |
| pico= <u>0. 000000000001</u> | p      | -12 places       | 1p    |

Converting **kWh** (kilowatt hours) to Joules:

$$1 \text{ kW} = 1000 \text{ J/s} \quad 1 \text{ hour} = 3600 \text{ s}$$

$$1 \text{ kWh} = 1000 \times 3600$$

$$= 3.6 \times 10^6 \text{ J}$$

$$= 3.6 \text{ MJ}$$

Conversion of power into Ampere:

If you have a voltage of 220V and a power of 220W, we can calculate the current using the formula for power:

$$P = V \times I$$

$$220 \text{ W} = 220 \text{ V} \times I$$

Dividing both sides of the equation by 220V, we get:

$$I = 220 \text{ W} / 220 \text{ V}$$

$$I = 1 \text{ A}$$

Convert from electron volts to joules:

For example, how many joules are equivalent to 5 MeV

$$\text{Charge of one electron} = 1.6 \times 10^{-19} \text{ C}$$

$$E = QV$$

$$E = 1.6 \times 10^{-19} \times 1 \times 5 \times 10^6$$

$$E = 8.0 \times 10^{-13} \text{ J}$$

### 3.1.2 - Limitation of Physical Measurements:

#### Random Error:

“Unpredictable and uncontrollable variations in measurement or observation that lack a consistent pattern”.

Random mistakes have an impact on precision because they produce variances in measurements, resulting in a spread around the mean. It is impossible to eliminate all random errors.

**Electronic noise** in an electrical instrument's circuit is an example of random error.

- ✚ Take at least three repeats and compute a mean; this method also enables for the identification of abnormalities.
- ✚ To reduce human error and enable shorter intervals, use computers/data loggers/cameras.
- ✚ Use the proper equipment, for example, a micrometre has a higher resolution (0.1 mm) than a ruler (1 mm).

#### Systematic errors:

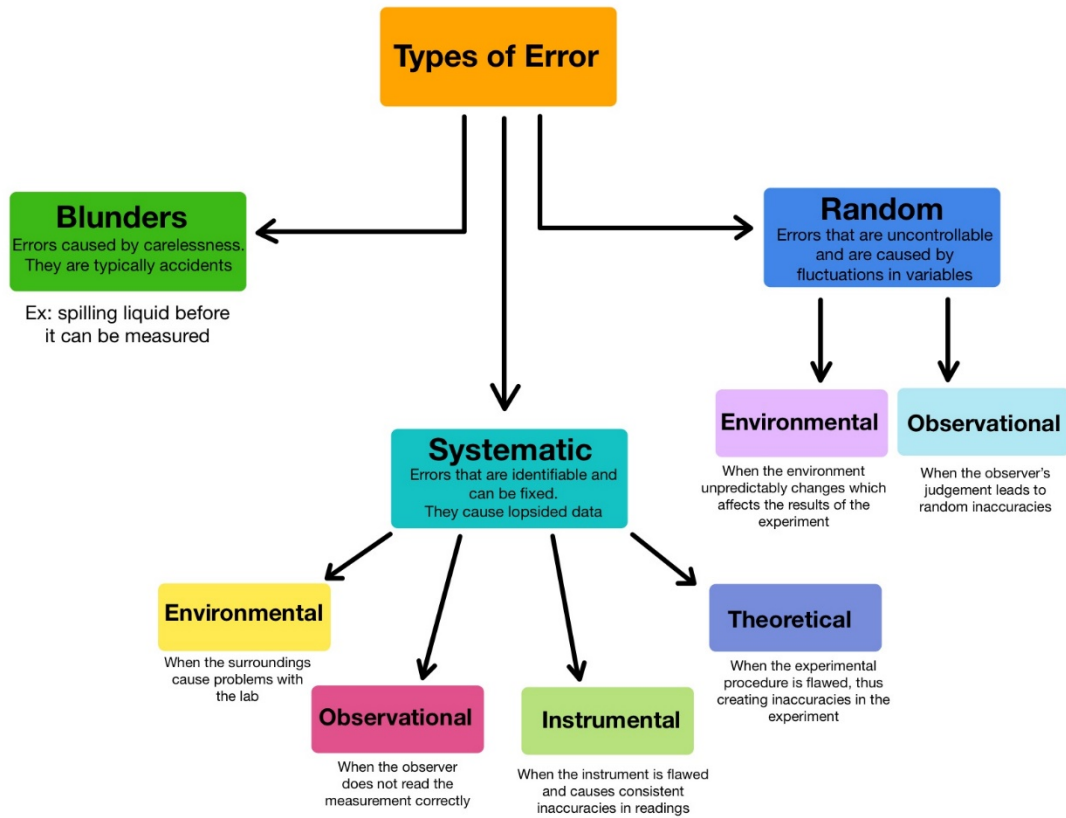
“Deviations from the true value in measurement or observation that are consistent and predictable.”

lower accuracy and are caused by errors in the experimental procedure or the device. Systematic mistakes lead all outcomes to be consistently too high or too low.

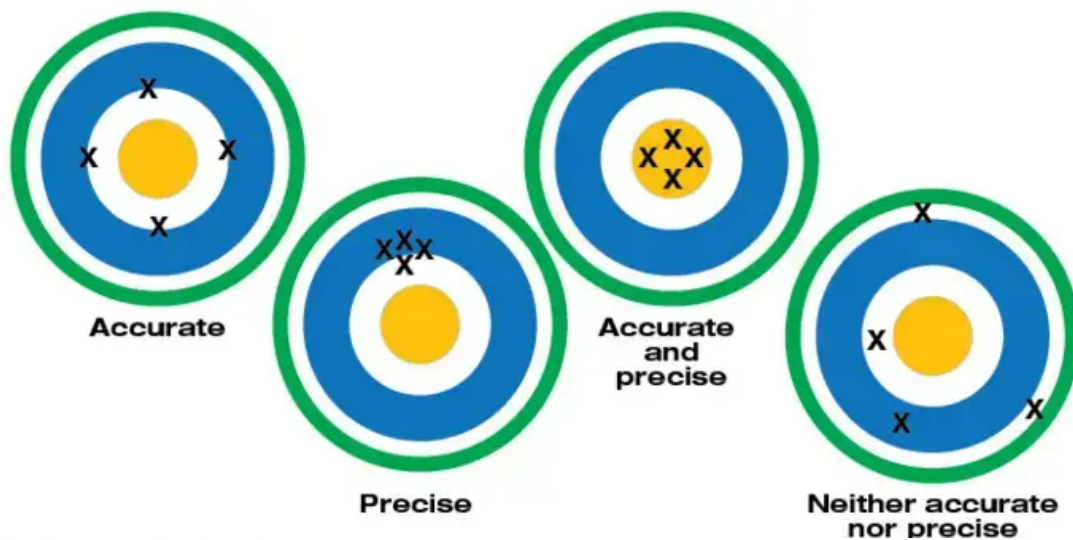
A balance that is not accurately zeroed (zero error) or reading a scale at an improper angle (parallax error) are two examples.

To reduce systematic error, do the following:

- ✚ Calibrate the device by measuring a known value (for example, weighing 1 kg on a mass balance); if the reading is incorrect the systematic error is easily identifiable.
- ✚ Correct for background radiation in radiation tests by detecting it beforehand and eliminating it from final results.
- ✚ To reduce parallax error, read the meniscus (the centre curve on the surface of a liquid) at eye level and utilise controls in tests.



**Precision and Accuracy:**



**Precision:**

Precise measurements are consistent; they vary slightly about a mean value; nevertheless, this does not mean that the value is accurate.

**Repeatability:**

It is reproducible if the original experimenter can conduct the experiment with the same equipment and approach and achieve the same findings.

**Reproducibility:**

It is reproducible if the experiment is repeated by a different individual or with various methods and equipment and the same findings are obtained.

**Resolution:**

The smallest change in the quantity being measured that results in noticeable variation in the reading

**Accuracy:**

A measurement that is near to the true value is accurate.

**Uncertainty:**

“The uncertainty of a measurement is the range within which the accurate value can be expected to occur.”



**Example:**  $22^{\circ}\text{C} \pm 2^{\circ}\text{C}$ , the true value could be between  $20^{\circ}\text{C}$  and  $24^{\circ}\text{C}$ .

## Absolute, Fractional, and Percent Uncertainty

| Absolute Uncertainty   | Fractional Uncertainty  | Percent Uncertainty  |
|--|---|--|
| Uncertainty expressed as a number independent of the original number | Uncertainty expressed as a fraction of the original number  | Uncertainty expressed as a percentage of the original number.  |
| Symbol:<br>$\Delta a$ or $\Delta b$ or $\Delta c$ etc.               | Symbol (and equation):<br>$\frac{\Delta a}{a}$ or $\frac{\Delta b}{b}$ or $\frac{\Delta c}{c}$ etc. | Symbol (and equation):<br>$\frac{\Delta a}{a} \times 100\%$ or $\frac{\Delta b}{b} \times 100\%$ or $\frac{\Delta c}{c} \times 100\%$ etc. |
| Example:<br>$4 \pm 1$  | Example:<br>$4 \pm 0.25$  | Example:<br>$4 \pm 25\%$   |

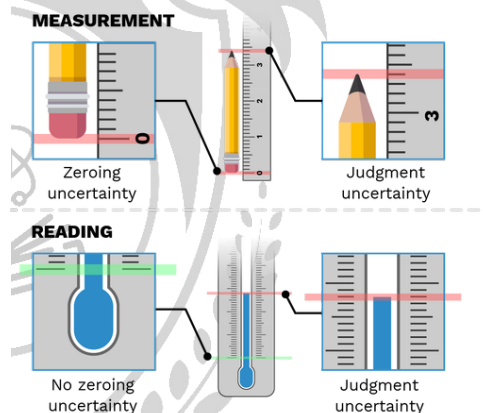
More

quantities can be measured to reduce percentage and fractional uncertainty.

## Resolution and Uncertainty:

### Reading vs Measurement

- For some measuring instruments, the reading you get from the instrument is the measurement of the quantity (e.g. thermometer, measuring cylinder)
- For some measuring instruments, there is a difference between the reading on an instrument, and the actual measurement



**The uncertainty in a reading** is half the smallest division, for example,  $1^{\circ}\text{C}$  for a thermometer, so the uncertainty is  $0.5^{\circ}\text{C}$ .

**The uncertainty in a measurement** is at least one smallest division, for example, a ruler, must include both the uncertainty for the start and finish value, since each end has  $0.5\text{mm}$ , they are added so the uncertainty in the measurement is  $1\text{mm}$ .

### Digital readings

The uncertainty in digital readings and given values will be expressed or presumed to be the final significant digit, e.g. The resolution of an instrument,  $3.2 \pm 0.1\text{ V}$ , influences its uncertainty.

For repeated data, the uncertainty is half the range (biggest - lowest value), which is represented mean  $\pm$  range/2.

Fixing one end of a ruler reduces uncertainty because only the uncertainty in one reading is included. You can also reduce uncertainty by measuring numerous occurrences,

**For example**, to get the time for one swing of a pendulum, measure the time for ten, providing  $6.2 \pm 0.1$  s, the time for one swing is  $6.2 \pm 0.1$  s, (the uncertainty is also divided by ten).

The same number of significant numbers as the data should be assigned to uncertainties.

### Combining uncertainties:

#### Adding / subtracting data - ADD ABSOLUTE UNCERTAINTIES:

##### Steps:

- Write down the values and their respective absolute uncertainties.
- Round the final result to an appropriate number of significant figures based on the Perform the addition or subtraction operation as normal.
- Add the absolute uncertainties of the values being combined.
- uncertainties involved.

Addition:

Measurement 1: Value =  $8.2 \pm 0.1$

Measurement 2: Value =  $3.7 \pm 0.2$

To add these measurements, you would perform the addition as follows:

$$8.2 + 3.7 = 11.9$$

To calculate the absolute uncertainty of the result, add the absolute uncertainties:

$$0.1 + 0.2 = 0.3$$

Therefore, the sum of the measurements is  $11.9 \pm 0.3$ .

Subtraction:

Measurement 1: Value =  $15.6 \pm 0.4$

Measurement 2: Value =  $9.3 \pm 0.3$

To subtract these measurements, you would perform the subtraction as follows:

$$15.6 - 9.3 = 6.3$$

To calculate the absolute uncertainty of the result, add the absolute uncertainties:

$$0.4 + 0.3 = 0.7$$

Therefore, the difference of the measurements is  $6.3 \pm 0.7$ .

Remember, these examples assume that the uncertainties are independent and random.

**For example:** If a thermometer with an uncertainty of  $\pm 0.5$  K shows the temperature of water falling from  $298 \pm 0.5$  K to  $273 \pm 0.5$  K, what is the temperature difference?

$$298 - 273 = 25\text{K}$$

$$0.5 + 0.5 = 1\text{K (Add absolute uncertainty)}$$

A difference of  $25 \pm 1$  K.

### **Multiplying / dividing data - ADD PERCENTAGE UNCERTAINTIES:**

Here are examples of multiplication and division involving two data points with their respective percentage uncertainties:

#### **Steps:**

- Write down the values and their respective percentage uncertainties.
- Perform the multiplication or division operation as normal.
- Add the percentage uncertainties of the values being combined.
- Round the final result to an appropriate number of significant figures based on the uncertainties involved

Multiplication:

Measurement 1: Value =  $8.2 \pm 2\%$  (percentage uncertainty)

Measurement 2: Value =  $3.7 \pm 1\%$  (percentage uncertainty)

To multiply these measurements, you would perform the multiplication as follows:

$$8.2 * 3.7 = 30.34$$

To calculate the percentage uncertainty of the result, add the percentage uncertainties:

$$2\% + 1\% = 3\%$$

To obtain the absolute uncertainty, multiply the result by the percentage uncertainty and divide by 100:

$$30.34 * (3/100) = 0.9102$$

Therefore, the product of the measurements is  $30.34 \pm 0.9102$ .

Division:

Measurement 1: Value =  $15.6 \pm 1\%$  (percentage uncertainty)

Measurement 2: Value =  $9.3 \pm 2\%$  (percentage uncertainty)

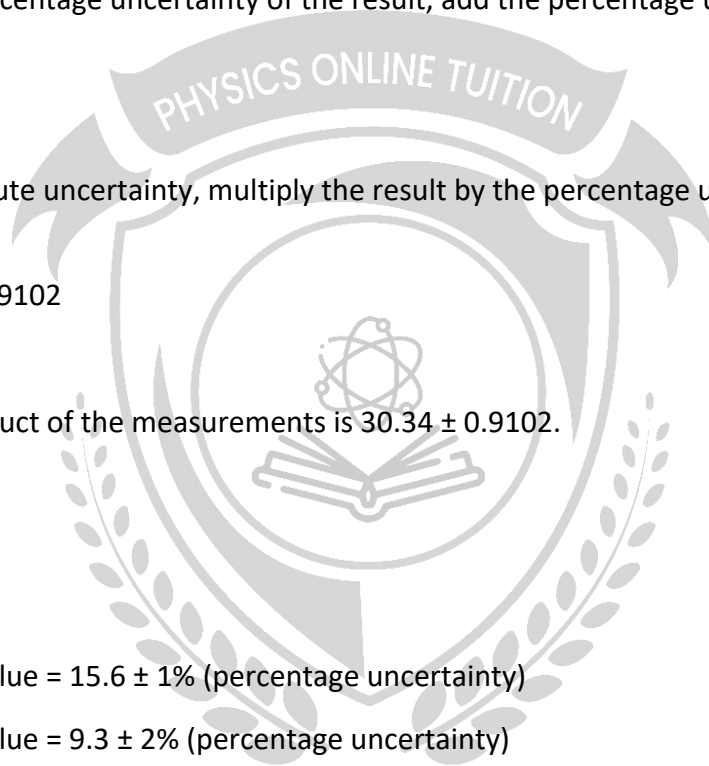
To divide these measurements, you would perform the division as follows:

$$15.6 / 9.3 = 1.6774$$

To calculate the percentage uncertainty of the result, add the percentage uncertainties:

$$1\% + 2\% = 3\%$$

To obtain the absolute uncertainty, multiply the result by the percentage uncertainty and divide by 100:



$$1.6774 * (3/100) = 0.05032$$

Therefore, the quotient of the measurements is  $1.6774 \pm 0.05032$ .

Remember, these examples assume that the uncertainties are independent and random.

**For Example:** a force of  $91 \pm 3$  N is applied to a mass of  $7 \pm 0.2$  kg, what is the acceleration of the mass?

$$a = F/m = 91/7 = 13 \text{ m s}^{-2} \qquad \text{percentage uncertainty} = \text{uncertainty}/\text{value} * 100$$

$$\text{Work out \% uncertainties } 3/91 \times 100 + 0.2/7 \times 100 = 3.3\% + 2.9\%$$

$$\text{add \% uncertainties} = 6.2\%$$

$$a = 13 \pm 6.2\% \text{ ms}^{-2}$$

$$a = 13 \pm 0.8 \text{ ms}^{-2}$$

Hint;

$$13 \times 6.2/100 \\ = 0.8$$

### Raising to a power - MULTIPLY PERCENTAGE UNCERTAINTY BY POWER:

#### Steps:

1. Write down the value you want to raise to a power, along with its percentage uncertainty.
2. Raise the value to the desired power.
3. Multiply the percentage uncertainty by the absolute value of the power.
4. Multiply the result of step 2 by the factor obtained in step 3.
5. Round the final result to an appropriate number of significant figures based on the uncertainties involved.

Let's illustrate this process with an example:

Given:

Value:  $7 \pm 2\%$  (percentage uncertainty)

Power:  $n = 3$

Write down the value and its percentage uncertainty:

Value = 7

Percentage Uncertainty = 2%

Raise the value to the power:

$$7^3 = 343$$

Multiply the percentage uncertainty by the absolute value of the power:

$$2\% * |3| = 6\%$$

Multiply the result of step 2 by the factor obtained in step 3:

$$343 * (6/100) = 20.58$$

Round the final result to an appropriate number of significant figures:

The result is 20.58, but considering the original uncertainty ( $\pm 2\%$ ), the final result is  $20.6 \pm 2\%$ .

Therefore, when raising the value  $7 \pm 2\%$  to the power of 3, the result is  $20.6 \pm 2\%$ .

**For Example:**

The radius of a circle is  $5 \pm 0.3$  cm, what is the percentage uncertainty in the area of the circle?

$$\text{Area} = \pi \times 25 = 78.5 \text{ cm}^2$$

$$\text{Area} = \pi r^2$$

$$\% \text{ uncertainty in radius} = 0.3/5 * 100 = 6\%$$

$$\% \text{ uncertainty in area} = 6 \times 2 \text{ (2 is the power from } r^2 \text{)}$$

$$= 12\%$$

$$= 78.5 \pm 12\% \text{ cm}^2$$

$$= 78.5 \pm 0.0001 \text{ cm}^2$$

Hint;

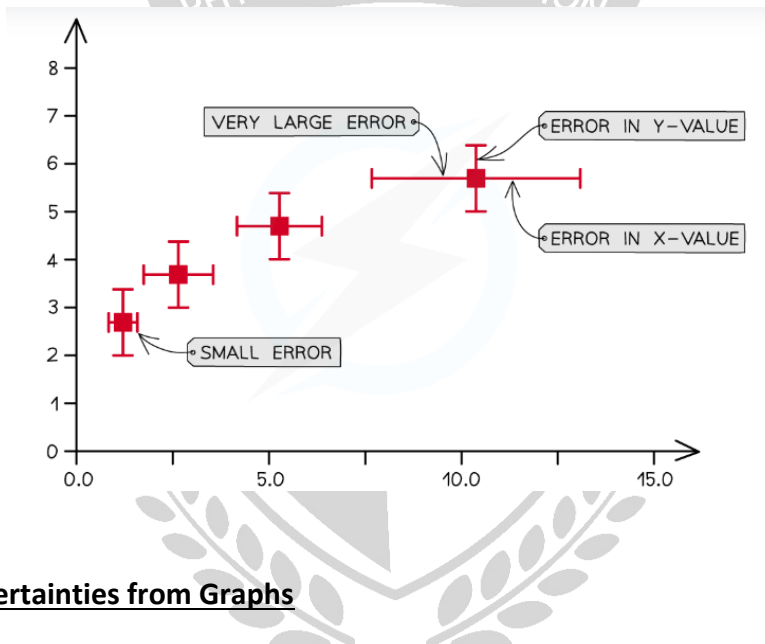
$$78.5 \times 12/100 \\ = 0.0001$$

## Uncertainties and graphs:

### Error Bar:

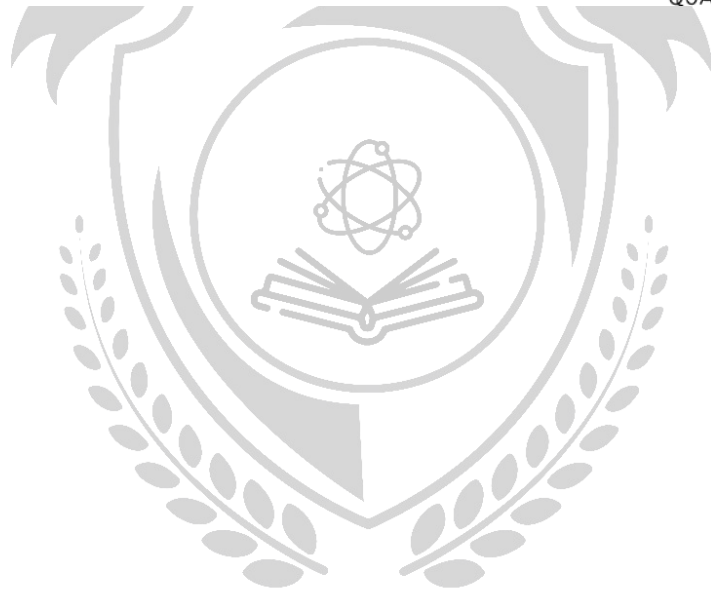
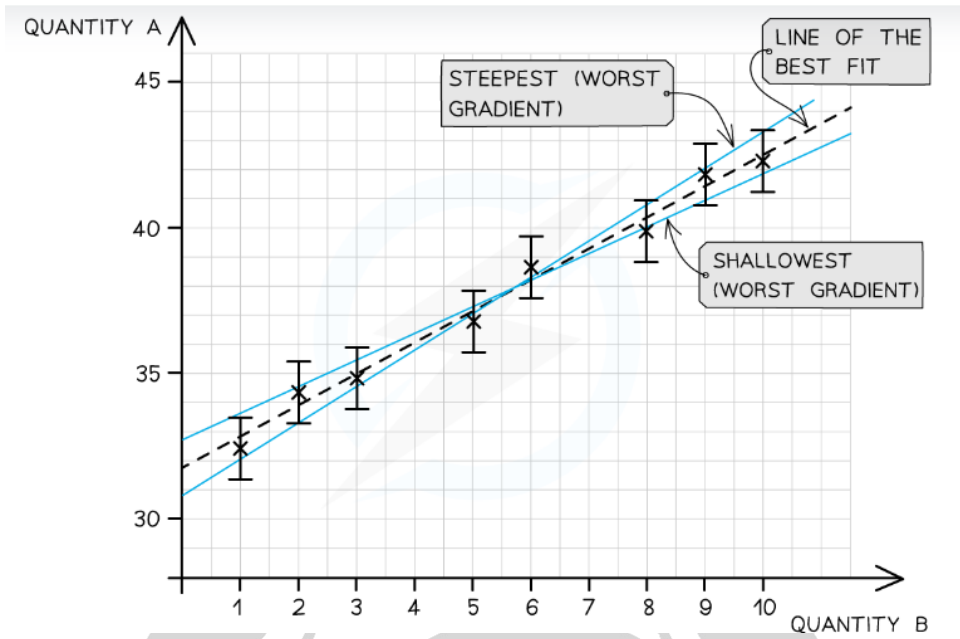
“An error bar can be used to represent the uncertainty in a measurement on a graph. This bar represents the uncertainty in that measurement and is drawn above and below the spot (or from side to side)”

- The uncertainty in a measurement can be shown on a graph as an **error bar**
- This bar is drawn above and below the point (or from side to side) and shows the **uncertainty** in that measurement
- Error bars are plotted on graphs to show the **absolute uncertainty** of values plotted



### Determining Uncertainties from Graphs

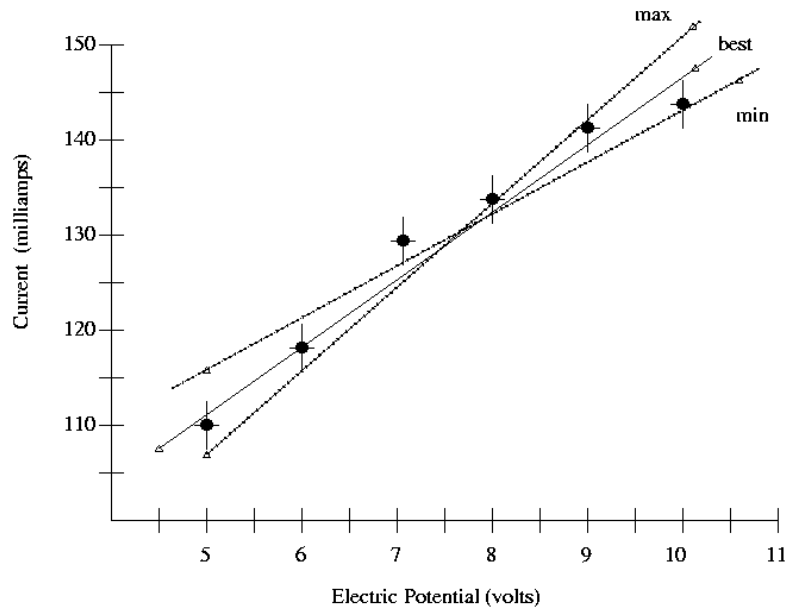
- To calculate the **uncertainty in a gradient**, two lines of best fit should be drawn on the graph:
  - The ‘best’ line of best fit, which passes as **close** to the points as **possible**
  - The ‘worst’ line of best fit, either the **steepest possible** or the **shallowest possible** line which fits within all the error bars



**For Example:**



If there are several points on a graph, the uncertainty in the slope should be calculated as follows. The data points are shown in the image below by small, full black circles; each datum has error bars to highlight the uncertainty in each measurement. Current appears to be measured to  $\pm 2.5$  milliamps and voltage to  $\pm 0.1$  volts. The hollow triangles represent slope-calculating locations. Take note of how I calculated the slopes by selecting places towards the ends of the lines.



1. Draw the "best" line through all the points, taking into account the error bars. Measure the slope of this line.
2. Draw the "min" line -- the one with as small a slope as you think reasonable (taking into account error bars), while still doing a fair job of representing all the data. Measure the slope of this line.
3. Draw the "max" line -- the one with as large a slope as you think reasonable (taking into account error bars), while still doing a fair job of representing all the data. Measure the slope of this line.
4. Calculate the uncertainty in the slope as one-half of the difference between max and min slopes.

In the example above, I find

$$\text{"best" slope} = \frac{147 \text{ mA} - 107 \text{ mA}}{10 \text{ V} - 4.5 \text{ V}} = 7.27 \text{ mA V}^{-1}$$

$$\text{"min" slope} = \frac{145 \text{ mA} - 115 \text{ mA}}{10.5 \text{ V} - 5.0 \text{ V}} = 5.45 \text{ mA V}^{-1}$$

$$\text{"max" slope} = \frac{152 \text{ mA} - 106 \text{ mA}}{10 \text{ V} - 5.0 \text{ V}} = 9.20 \text{ mA V}^{-1}$$

$$\text{Uncertainty in slope is } 0.5 * (9.20 - 5.45) = 1.875 \text{ mAV}^{-1}$$

There are at most two significant digits in the slope, based on the uncertainty. So, I would say the graph shows

$$\text{slope} = 7.3 \pm 1.9 \text{ mAV}^{-1}$$

### 3.1.3 - Estimation of physical quantities:

#### Orders of magnitude:

Powers of ten used to define the size of an object and to compare the sizes of objects.

E.g: The diameter of nuclei is approximately  $10^{-14}$  m in order of magnitude.

100 metres is two orders of magnitude greater than one metre.

You may be asked to offer a value to the nearest order of magnitude; in this case, simply compute the value requested and give it only as a power of ten.

E.g Find the estimated area of the entire atom (assuming it is perfectly spherical) to the nearest order of magnitude if the diameter of a hydrogen atom is  $1.06 \times 10^{-10}$  m

Find the area using  $A = \pi r^2$

$$\pi \times (0.53 \times 10^{-10})^2 = 8.82 \times 10^{-21} \text{ m}^2 = 1 \times 10^{-20} \text{ m}^2 \text{ (to 1 s.f)}$$

Therefore the area to the nearest order of magnitude is  $10^{-20}$  m<sup>2</sup>.

**Estimation is a skill that physicists must master in order to approximate the values of physical quantities, make comparisons, or determine whether a number they calculated is appropriate.**