FURTHER MECHANICS

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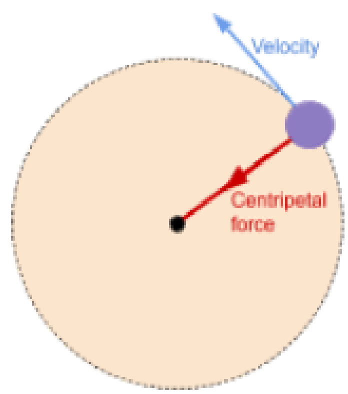
Circular Motion:

Imagine a car going around a roundabout. Even if it's going at a constant speed, it's always changing its direction, right? That's because its velocity includes both how fast it's going (speed) and where it's going (direction). To keep going in a circle, the car needs to keep turning, which means it's accelerating, and it needs a force to do that. This force that keeps it going in a circle is called the centripetal force, and it always points toward the center of the circle.

We know from Newton's first law that to accelerate, an object must experience a resultant force, therefore an object moving in a circle must experience a force, this is known as the centripetal force, and it always acts towards the centre of the circle.



Angular Speed (ω):

This is like measuring how fast something is spinning or going around in a circle. To figure it out, you can take the object's speed and divide it by how far it is from the center of the circle. It's a way to measure how quickly the car is spinning around the roundabout.

"Angular speed (ω) is the angle an object moves through per unit time"

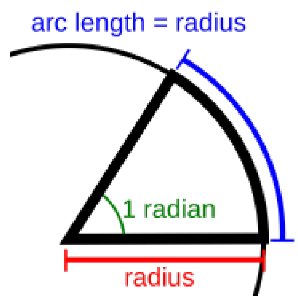
$$\omega = \frac{v}{R}$$

$$\frac{2\pi}{T} = 2\pi f$$

as $f = \frac{1}{T}$

Radians:

Radians are just a different way to measure angles. One radian is the angle made when you take a slice of a pizza that has the same length as the radius of the whole pizza. If you go all the way around a circle, that's 2π radians (about 6.28 radians). You can convert between degrees and radians by multiplying by a number (π for going from degrees to radians and 180 for going from radians to degrees).



Angles can be measured in units called radians. One radian is defined as the angle in the sector of a circle when the arc length of that sector is equal to the radius of the circle.

Considering a complete circle, its arc length is $2\pi r$, dividing this by r, you get 2π which is the angle in radians of a full circle. From this you can convert any angle from degrees to radians by multiplying by $\pi/180$ and from radians to degrees by multiplying by $180/\pi$.

So, when things go around in circles, they're not just moving at a constant speed; they're also changing their direction, and they need a force to keep them on track. We use angular speed and radians to help us understand how fast they're spinning or moving in circles.

Centripetal Acceleration (a):

This is the force that keeps things moving in a circle. Imagine spinning a yo-yo on a string. The yo-yo is always trying to fly off in a straight line, but the string pulls it in, making it change direction and stay in a circle. That change in direction is acceleration. You can find this acceleration using the formula $a = v^2/r$, which means it depends on how fast the yo-yo is spinning (v) and how far it is from the center (r).

$$a = \frac{v^2}{r} = \omega^2 r$$

$$F = \frac{mv^2}{r} = m\omega^2 r$$

Centripetal Force (F):

To keep the yo-yo moving in a circle, you need to apply a force. This force is the centripetal force. It's like the pull of the string on the yo-yo. You can find it using $F = mv^2/r$, where m is the mass of the yo-yo.

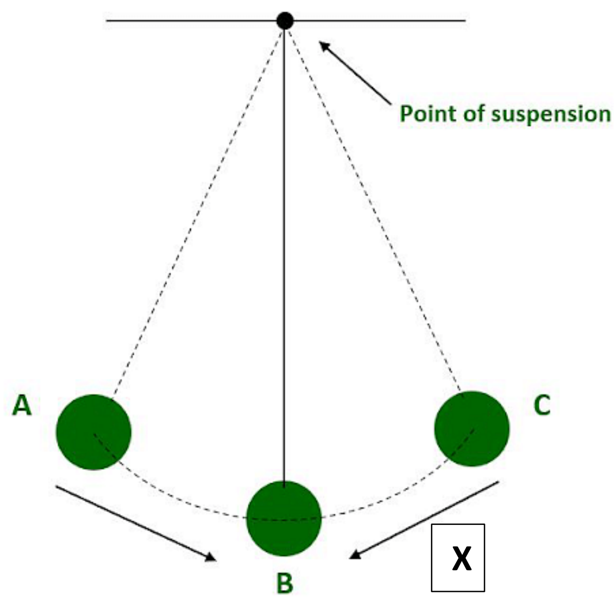
Simple Harmonic Motion (SHM):

This is like a back-and-forth or up-and-down motion that follows a pattern. A great example is a pendulum swinging back and forth. When you pull it to one side and let go, it swings to the other side and then back again, and this keeps repeating. The acceleration in SHM is directly proportional to how far it is from the center position (equilibrium) and in the opposite direction. You can describe this with the equation $a = -\omega^2 x$, where a is acceleration, ω is how fast it's swinging, and x is how far it is from the center.

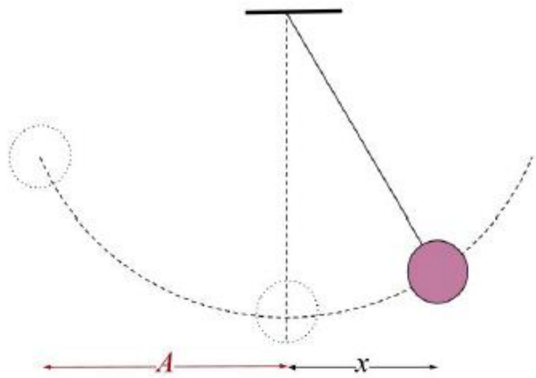
Simple harmonic motion when its acceleration is directly proportional to displacement and is in the opposite direction. These conditions can be shown through the equation:

$$a = -\omega^2 x$$

Where a is acceleration, ω is angular speed, x is displacement from the equilibrium position.



So, when things move in a circle, they experience centripetal acceleration, which keeps them on the circular path, and this requires a centripetal force. In simple harmonic motion, things move back and forth in a regular pattern, like a pendulum. The acceleration in SHM depends on how far from the center position it is and goes in the opposite direction. This helps us understand how things like swings, pendulums, or even sound waves work.

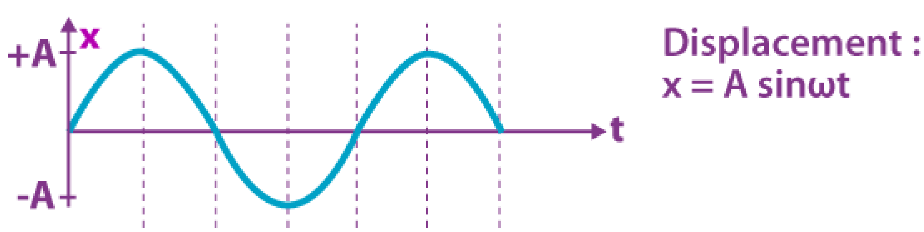


x is the measure of **displacement**, and by an A is the **amplitude** of the oscillations, this is the maximum displacement.

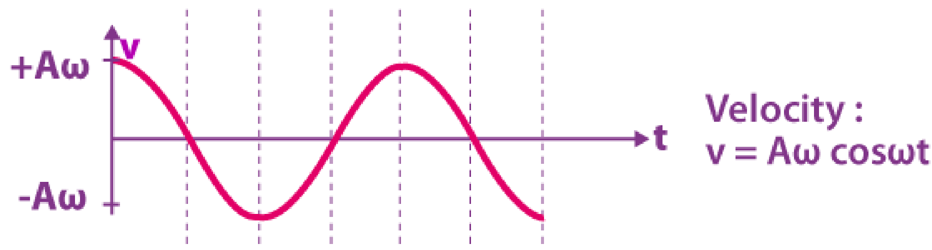
$$x = A\cos\omega t$$

$$v = \pm\omega\sqrt{(A^2 - x^2)}$$

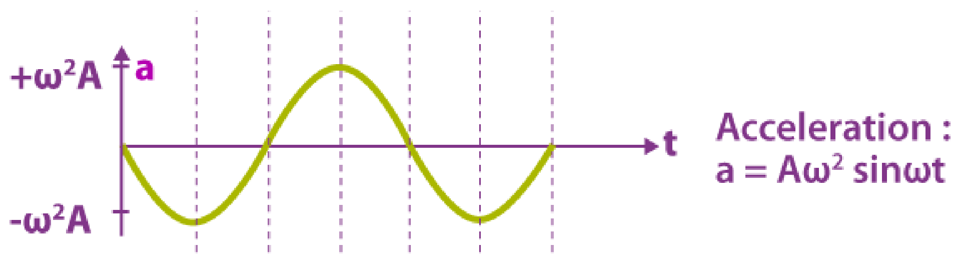
From the above equation above the displacement-time graph will follow a cosine or sine curve, with a maximum A, and minimum -A because A and ω are constants:



Velocity is the derivative of displacement, we can draw a **velocity-time graph**by drawing the **gradient function of the above graph**, noting that the maximum (ωA) and minimum velocity ($-\omega A$) occurs when x is 0, as expected from the above formula:



Acceleration is the derivative of velocity, so **acceleration-time graph** by drawing the **gradient function of the above graph**.



Simple Pendulum:

Imagine you have a heavy weight (the bob) hanging from a string. When you pull the bob to one side and then let it go, it swings back and forth like a pendulum. If you only pull it a little bit (less than 10 degrees), it moves in a predictable way, like a metronome. The time it takes to swing back and forth once is called the time period (T), and you can find it using the formula $T = 2\pi\sqrt{(l/g)}$, where l is the length of the string and g is the force of gravity. The reason you need to keep the initial angle small is that this formula only works well for small angles.

As the pendulum swings, it keeps switching between two types of energy: gravitational potential energy (when it's high up) and kinetic energy (when it's moving fast). It's like a rollercoaster, going up and down, changing between the energy of height and the energy of motion.

- **Simple pendulum - A small, dense bob of mass m hangs from a string of length l , which is attached to a fixed point. When the bob is displaced by a small angle (less than 10°), and let go it will oscillate with SHM. For this type of system, you can use the following formula:**

$T = 2\pi\sqrt{l/g}$

where T is time period, l is the length of the string, g is acceleration due to gravity

The reason the angle by which the pendulum is displaced must be less than 10°, is because during the derivation of the above formula a small angle approximation is used, and so for larger initial angles this approximation is no longer valid, and would not be a good model.

During the oscillations of a simple pendulum, its gravitational potential energy is transferred to kinetic energy and then back to gravitational potential energy and so on.

MASS- SPRING SYSTEM

This is like a weight (mass) attached to a spring. When you pull the mass down and let it go, it bounces up and down. There are two types: one where the spring hangs vertically, and the other where it stretches horizontally. In the vertical system, the mass bounces, and it's also affected by gravity, so it changes between elastic potential energy (from the spring) and gravitational potential energy (from being high up). In the horizontal system, it only switches between elastic potential energy and kinetic energy, as there's no gravity pulling it down.

In both cases, you can find the time period (T) using the formula T = 2π√(k/m), where k is the spring constant, and m is the mass of the weight.

For both simple pendulums and mass-spring systems, the energy keeps going back and forth between potential energy (like a coiled spring ready to jump) and kinetic energy (like a car racing down a hill). This energy switching happens over and over as the object moves back and forth. If there's no air resistance, the total energy of the system stays the same. If you've ever seen a pendulum or a spring bounce up and down, you've seen this energy conversion in action

The mass-spring system is another classic example of simple harmonic motion, where a mass is attached to a spring and undergoes oscillations. The behavior of the system depends on whether the spring is arranged vertically or horizontally.

Vertical Mass-Spring System:
In a vertical mass-spring system, when the mass is displaced from its equilibrium position and released, it oscillates up and down. As the mass moves upward, it gains potential energy due to the stretching of the spring and also gains gravitational potential energy. As it moves downward, these potential energies are converted back to kinetic energy. The formula you provided for the time period in this case is:

$$T = 2\pi\sqrt{k/m}$$

where T is time period, m is the mass, k is the spring constant

Where:

T is the time period,
m is the mass,
k is the spring constant.

Horizontal Mass-Spring System:
In a horizontal mass-spring system, the mass oscillates back and forth along a horizontal axis. In this case, only elastic potential energy is involved, as there is no change in gravitational potential energy. The formula for the time period (T) in the horizontal system is the same as for the vertical system:

This is because the time period of oscillation in a mass-spring system is determined by the mass (m) and the spring constant (k).

$T = 2\pi\sqrt{k/m}$

where T is time period, m is the mass, k is the spring constant

In both cases, the system undergoes simple harmonic motion, and the time period is independent of the amplitude of the oscillations. The energy exchange between kinetic and potential energy is a characteristic feature of simple harmonic motion, and the total mechanical energy remains constant if there are no external damping forces

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Potential and Kinetic Energy Exchange:

At the maximum displacement (amplitude), the system has maximum potential energy, whether it's gravitational potential energy in the case of a vertical mass-spring system or elastic potential energy in the case of a horizontal mass-spring system or a simple pendulum.

As the system moves towards the equilibrium position, potential energy is gradually converted to kinetic energy.

Equilibrium Position:

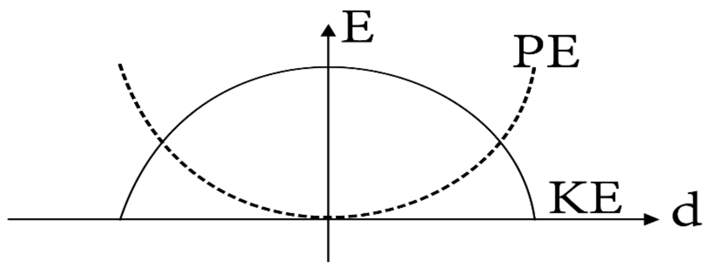
At the equilibrium position (the center of oscillation), the kinetic energy is at its maximum because potential energy is zero.

Energy Transfer during Oscillation:

As the system moves away from the equilibrium position, kinetic energy is transferred back to potential energy. The nature of potential energy depends on the specific type of system (gravitational potential energy in a vertical mass-spring system or elastic potential energy in a horizontal mass-spring system).

Total Energy Conservation:

In the absence of damping forces (like air resistance), the total mechanical energy of the system remains constant throughout the oscillation. The sum of kinetic and potential energy is conserved, and the system perpetually oscillates back and forth.



Light Damping (Under-damping):

- Characterized by a gradual decrease in amplitude with each oscillation.
- The system takes a longer time to return to its equilibrium position.
- Oscillations persist for some time before coming to a stop.
- Commonly observed in systems with relatively low damping forces.

Critical Damping:

- Results in the quickest return to the equilibrium position without any oscillations.
- Amplitude decreases to zero in the shortest possible time.
- Achieved when the damping force is at a specific critical value.
- It's the ideal scenario for applications where a rapid return to equilibrium is crucial.

Heavy Damping (Over-damping):

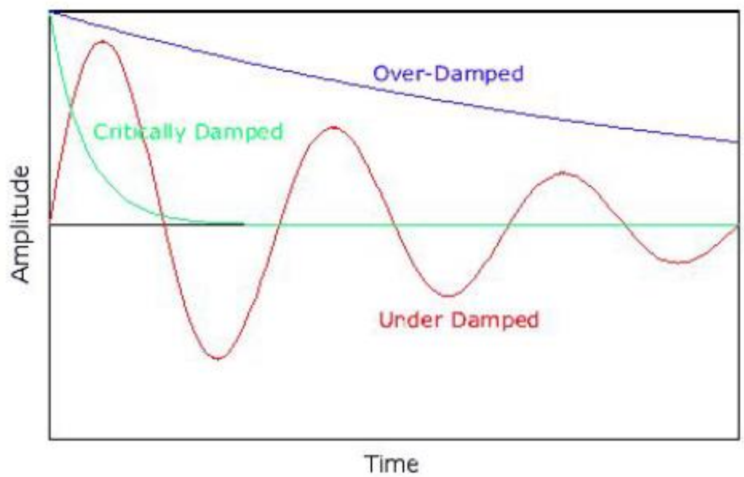
- Amplitude decreases more slowly compared to critical damping.
- Similar to under-damping, there are no additional oscillations.
- The system takes a longer time to return to equilibrium compared to critical damping.
- Commonly encountered in systems with high damping forces.
- The relationship between displacement and energy, as well as time and energy, can provide insights into the behavior of the system under different damping conditions. For example, critical damping minimizes the time taken to return to equilibrium, while heavy damping sacrifices speed for stability.

Free Vibrations:

These occur when no external force continuously acts on a system, allowing it to oscillate at its natural frequency.

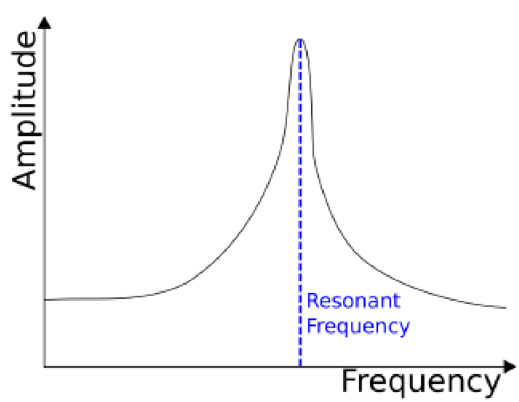
Forced Vibrations:

External forces drive the system, and the frequency of this force (driving frequency) is crucial. If it equals the natural frequency, resonance occurs.



Resonance:

The amplitude of oscillations significantly increases due to the system gaining energy from the driving force. Resonance has practical applications, such as in musical instruments, radios.



Applications of Resonance:

Instruments: Instruments like flutes use resonance to create stationary sound waves.

Radio: Radios are tuned to resonate at the desired broadcast frequency.

Swings: Pushing someone on a swing provides a driving frequency, potentially causing resonance and higher swinging.

Negative Consequences of Resonance:

Resonance can have adverse effects, leading to structural damage. For example, a bridge may oscillate violently if the frequency of people crossing it is close to the natural frequency.

Damping:

Damping can be employed to mitigate the effects of resonance. Different damping types have varying effects on resonant frequency, amplitude, and peak amplitude width. Increased damping results in a lower resonant frequency, decreased maximum amplitude, and a wider peak.